SPARSE VECTOR METHOD BASED CONTINGENCY EVALUATION

by

CHANDRA SHEKHAR TIWARI

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

APRIL, 1986

SPARSE VECTOR METHOD BASED CONTINGENCY EVALUATION

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

by

CHANDRA SHEKHAR TIWARI

to the

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

APRIL, 1986

DEDICATED TO

MY MOTHER

AND

LATE FATHER

CERTIFICATE

It is to certify that this work entitled 'SPARSE VECTOR METHOD BASED CONTINGENCY EVALUATION' by CHANDRA SHEKHAR TIWARI has been carried out under my supervision and that this work has not been submitted elsewhere for the award of a degree.

Sachchidanand

(Sachchidanand)
Assistant Professor
Department of Electrical Engineering
Indian Institute of Technology
KANPUR

ACKNOWLEDGEMENTS

It is with a profound sense of gratitude, I express my indebtedness to my thesis supervisor, Dr. Sachchidanand, whose active guidance and constant inspiration has been greatly instrumental in the completion of this thesis.

I am also very grateful to Dr. L.P. Singh for his valuable advice and guidance throughout the duration of the M.Tech. course.

I would like to express my sincere thanks to all my friends and colleagues and in particular to Senthil, Nirmal, Poonacha, Rajiv, Capt. Bhatia, Shri Kehar Singh, Shri Chaudhary, Shri PN Shreedhar and Shri K.K. Ghose, for their generous assistance and suggestions during the course of the work.

I am thankful to Shri Rawat for excellent typing. Thanks are also due to Shri Triveni Tiwari and Shri Ganga Ram for quick cyclostyling.

Last, but not the least, I am indebted to my wife for her patience and unceasing encouragement.

CHANDRA SHEKHAR TIWARI

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

This thesis reports on the investigation carried out to study the effectiveness of sparse vector method in contingency evaluation. The sparse vector method for solution of network equations has been examined in considerable detail, highlighting the scope for possible application. The sparse vector method has been tested for its viability in contingency evaluation based on Fast Decoupled Load Flow technique. Different sparsity oriented compensation methods have been reviewed and applied to contingency evaluation with a view to determining the suitability of sparse vector approach.

CHAPTER 1

INTRODUCTION

1.1 GENERAL:

Modern day power systems are all generally large and complex. Planning and operation of these systems require a number of studies to be carried out both in the off-line mode and on the real time basis. Most of these system studies necessitate solution of network equations involving large sparse matrices, which characterise the system. Sparse matrix methods [1,2] have been widely used for the solution of network equations, particularly from the view point of reducing computer storage requirement. However, for the system studies, to be carried out on real time basis, like contingency evaluation to ensure system security, it is important that the solution techniques be computationally efficient. In this context, solution techniques exploiting vector sparsity have been reported in the literature [3], both for planning and operation studies.

1.2 SPARSE VECTORS:

A system of linear algebraic equations may be represented as

$$[A]\ \underline{X} = \underline{B} \tag{1.1}$$

where,

[A] is the system matrix

$\underline{X}$ and $\underline{B}$ are vectors

In Eqn. (1.1), vector X is commonly referred to as the solution vector, while the right hand side vector B is called the independent vector. In certain power system studies, either the independent vector B may be sparse in structure or the interest may be only in determining certain components of the solution vector X. In such a situation, vector B or vector X are termed as sparse vectors.

In the commonly used method for the solution of linear algebraic equations, using factors of sparse A matrix, the network solution is obtained by performing full forward and full backward operations on the nonzero elements. However, with sparse vectors it is possible to avoid forward and backward operations on all the nonzero elements involved in the factors of sparse A matrix. The latter is termed as the sparse vector solution approach.

In recent years, attempt has been made to use sparse vector solution approach for various power system studies. Brandwajn and Tinney have applied the sparse vector approach to advantage in fault analysis [4]. The effectiveness

of this approach has also been investigated in case of sparsity oriented compensation methods by Alsac et.al. [5]. Possible application of sparse vector method in optimal power flow and economic despatch has been described in [6] and [7] respectively.

Besides, certain other areas have also been identified for the application of sparse vector methods. These include contingency evaluation or outage studies, partial refactorization, parallel processing and network equivalencing [3].

1.3 CONTINGENCY ANALYSIS:

Contingency evaluation appears attractive from the point of view of application of sparse vector method . Contingency evaluation which is a key function of security analysis is under taken on real time basis to analyse the outcome of contingencies such as line or transformer outages or loss of generation etc. on the security of the system.

A number of fast-on line approximate algorithms [8,9-11] have been proposed for contingency evaluation. The techniques used for contingency evaluation must possess the essential features of fast computational speed, reliability, reasonable accuracy and moderate memory storage requirements. The present trend in contingency evaluation algorithms is to

compensate the base case load flow results, using certain matrix modifications which simulate the effect of outages. This approach is preferred compared to the refactorization approach in which the base case system matrix is modified and then refactorized to obtain the network solution for the post-contingency operating state [5].

It is essential to highlight that in contingency evaluation, matrix modifications to reflect the configuration changes arising due to outages are derived on the basis of well known Inverse Matrix Modification Lemma [5,12,13].Since these methods of matrix modifications, offer a basis of providing compensation for network changes, these are generally referred to as compensation methods. A wide range of compensation methods has been reported in the literature [2,5]. It is in these compensation methods for contingency evaluation that sparse vector method may find possible application.

1.4 OBJECTIVE AND SCOPE OF THE THESIS:

From the review of available literature, it seems important to further explore the potential of sparse vector approach as regards its application, merits and limitations. The objectives of this thesis, thus, are:

a) To study the characteristics of sparse vectors and to examine the sparse vector method in detail with particular reference to its implementation for solution of linear algebraic equations.

b) To investigate the possible application of sparse vector method for contingency evaluation employing compensation techniques.

1.5 OUTLINE OF THE THESIS:

Chapter 2 deals with the study of sparse vectors as regards their characteristics. The derivation of sparse vector method for solution of network equations and its implementing algorithms are described. Further, the efficiency of sparse vector approach is examined in obtaining solutions of linear algebraic equations by conducting several case studies.

The application of sparse vector solution method for contingency evaluation is described in Chapter 3. The contingency evaluation uses compensation technique, to simulate the effects of outages. The effectiveness of the sparse vector approach is investigated through case studies on a number of sample power systems.

With reference to contingency analysis a comparative evaluation of various compensation techniques, considering the application of sparse vector method, is reported in Chapter 4. The computational requirements of various compensation techniques are compared through case studies on different sample power systems.

Chapter 5 reviews the major contribution of the thesis and suggests the scope for future work.

CHAPTER 2

SPARSE VECTOR METHOD

2.1 GENERAL:

While the sparse matrix methods for the solution of large scale power system problems are well established, vector sparsity has not been fully exploited despite its advantages. Tinney et.al. [3] have highlighted the usefulness and potential of the sparse vector approach for the solution of linear algebraic equations. An attempt has, therefore, been made in this chapter, to study the effectiveness of sparse vector methods in the solution of linear algebraic equations.

2.2 SPARSE VECTOR APPROACH:

Consider a system of linear algebraic equations

$$[A]\ \underline{X} = \underline{B} \qquad (2.1)$$

where

- $[A]$ is the system matrix of order N
- $\underline{X}$ is the vector of variables or solution vector
- $\underline{B}$ is the independent vector

Sparse vector methods require the system matrix [A] to be factored as

$$[A] = [L]\ [D]\ [U] \qquad (2.2)$$

where,

$[L]$ is the lower triangular matrix with unity diagonal

$[D]$ is the diagonal matrix and

$[U]$ is the upper triangular matrix with unity diagonal.

However, sparse vector methods are equally applicable to the cases where $[A]$ is factored as

$$[A] = [L]\,[U] \tag{2.3}$$

The matrix U is the transpose of matrix L when $[A]$ is symmetric, which is normally the case in respect of power system admittance matrices.

The solution vector X using triangular factorization can now be obtained from Eqn. (2.1) and Eqn. (2.2) as

$$[L]\,[D]\,[U]\,\underline{X} = \underline{B} \tag{2.4}$$

Substituting $[U]\,\underline{X} = \underline{Y}$ and $[L]\,[D] = [L']$, equation (2.4) gives

$$[L']\,\underline{Y} = \underline{B} \tag{2.5}$$

or

$$\underline{Y} = [L']^{-1}\,\underline{B} \tag{2.6}$$

Solution of Eqn. (2.5) gives an intermediate vector Y, which in turn is used to determine the solution vector X from

$$[U]\,\underline{X} = \underline{Y} \tag{2.7}$$

Then,

$$\underline{X} = [U]^{-1} \underline{Y} \tag{2.8}$$

Equation (2.6) defines a sequence of forward substitution operations on [L'], while Eqn. (2.8) defines a sequence of backward substitution operations on [U]. The operations can be performed by rows or columns. An analysis of the algebra involved in the sparse vector approach will show that for sparse vector method , it is essential that the forward solution of equation (2.6) be performed by columns and in the backward solution of Eqn. (2.8), the elements of [U] be accessed row-wise [3].

2.2.1 Fast Forward/Fast Backward Operations:

When the independent vector B is full, the sequence of forward substitution operations on elements of [L'], to obtain intermediate vector Y is required to be carried out for each column, in the ascending order of columns. This may be termed as the 'Full Forward Method'. If the vector B is sparse, only a subset of the columns of [L'] is needed for the forward solution of Eqn. (2.6). This is called the 'Fast Forward' (FF) solution.

Likewise, if all the elements of the solution vector X are required to be computed, it calls for 'Full Back'

substitution, in which case elements of each row of [U] are accessed. However, when the interest lies in knowing only certain elements of the vector X, then only a subset of rows of U is needed for the solution of Eqn. (2.8). This is termed as the 'Fast Backward' (FB) solution.

For the application of sparse vector method , it is necessary to efficiently identify the subsets of columns of [L'] and [U] for FF and FB solutions. The subset of columns for FF solution is a function of the sparsity structures of [L'] and $\underline{B}$. Similarly, the subset of rows for FB solution is a function of the sparsity structures of [U] and $\underline{X}$. The sparsity structures of [L'] and [U], in turn, are the functions of the sparsity structure of system matrix A.

Some algorithms have been proposed to identify the subset of columns of [L'] for Fast Forward solution and the subset of rows of [U] for Fast Backward solution [3]. These algorithms employ the concept of 'factorization paths', which enables identification of the relevant subset of columns and rows.

2.2.2 Factorization Paths:

Before defining a factorization path, it would be appropriate to describe a 'singleton', because the path

concept has been developed over the path associated with a singleton [3]. A vector with only one nonzero element may be referred to as a singleton. A 'factorization path' or 'path' can be defined as an ordered list of columns of [L'] for Fast Forward operations or rows of [U] for Fast Backward operations. The path for a singleton may be determined from the following algorithm.

a) Search for the nonzero element in the vector B and let k denote its location in the vector. Record k as the first element in the path.

b) Move to the column k of [L'] or row k of [U] and determine the location of the first nonzero element in column k of [L'] or row k of [U]. In searching for the first nonzero element, the diagonal elements of [L'] and [U] are not to be considered.

c) Replace the previous value of k in (a) with the location of the first nonzero element traced in (b) and list it in the path.

d) Repeat the steps (b) and (c) till k = N (N being the order of [L'] or [U]).

It needs to be mentioned that a path is executed in forward order for FF solution and in reverse order for FB solution.

The formation of the path associated with a singleton is illustrated through an example. Consider a 10 node network for which the structure of [L'] is shown in Fig. 2.1. While the star (*) signs in the figure indicate the locations of the nonzero elements initially present in [A], the ∞ signs represent the location of additional nonzero fill-in elements created in obtaining the factor [L']. Now, for a singleton B with nonzero in location 1, using the preceding algorithm, the path is obtained as 1,4,5,8,9,10.

NODE	1	2	3	4	5	6	7	8	9	10
1	*									
2		*								
3			*							
4	*			*						
5		*		*	*					
6		*				*				
7	*		*	∞			*			
8					*	*	∞	*		
9			*		∞		*	∞	*	
10		*				*			∞	*

* indicate the location of the nonzero elements initially present in [A].

∞ indicate the location of fill-in elements created in obtaining [L'].

Fig. 2.1: Structure of matrix L'

When matrix A is symmetric, its factors [L'] and [U] are also symmetric. In such a case, the path associated with a particular element in the vector X for FB solution would be the same as the path determined for FF solution, for the same location of nonzero element in vector B. Nevertheless, the subset of rows of [U] or path for FB substitution can be determined in an identical manner, using the preceding algorithm, irrespective of whether [L'] and [U] are symmetric or not. The sparsity structure for a non-symmetric matrix U is shown in Fig. 2.2. For a singleton with nonzero at location 2 i.e. in case the interest lies in obtaining only the second component of vector X, the path may be traced as 2,3,6,8,9,10 .

However, as mentioned earlier, it is essential that the path for Fast Backward solution be executed in reverse order implying that in case, the second component of vector X is required to be determined, then to use the Fast Backward solution, the path has to be executed in reverse order viz. 10,9,8,6,3,2 . If matrix A is stored exploiting sparsity through indexing arrays, the path for a singleton can also be determined directly from the indexing arrays, instead of searching and testing for the nonzero entry, as carried out in the above example, where the storage of full A matrix is assumed.

NODE	1	2	3	4	5	6	7	8	9	10
1	*						*		*	*
2		*	*							
3			*			*				
4				*			∞			
5					*			*		*
6						*		∞	*	
7							*			
8								*	*	
9									*	∞
10										*

Fig. 2.2: Sparsity structure of matrix U

In a general situation, it is possible that the vector B may have more than one nonzero elements or the interest may be in evaluating more than one component of the vector X. Corresponding to each nonzero entry in the vector B, a path can be obtained as explained earlier in case of a singleton. The union of these individual paths defines the complete factorization path for the sparse vector consisting of more

than one nonzero entry. The algorithm for obtaining the complete factorization path for a general sparse vector is described below [3].

i) Search for the first nonzero element in the vector B and trace its singleton path as explained earlier. This path segment should always end with node N.

ii) Identify the next nonzero node, which is not already in the partially completed path and determine its path in the same manner, until a node already in the partially completed path is encountered.

iii) Next, join this newly formed path segment to the partially completed path list.

iv) Steps (ii) and (iii) are repeated till all nodes in the sparse vector are in the path.

v) The path list is arranged either in ascending order for Fast Forward solution requirement or in reverse order for Fast Backward solution.

Based on the above algorithm, the determination of the complete factorization path for the FF solution is described for the sample 10 node system example. Consider a vector B with nonzero elements at location 1,3,6 and 7. Using step (i), the path corresponding to nonzero in location 1

for the sparsity structure of matrix L' represented in Fig. 2.1 is traced as 1,4,5,8,9,10 . The next nonzero being in location 3, the second path segment using step (ii) is obtained as 3,7 . Step (iii) gives the partially completed path as 3,7,1,4,5,8,9,10 . For the nonzero in location 6, the path segment is determined as 6 , comprising only one node. The partially completed path is then enlarged to give 6,3,7,1,4,5,8,9,10 . Path segment corresponding to nonzero in location 7 is not required to be traced, as node 7 already appears in the computed path list. The elements of the path list are then arranged in ascending order as 1,3,4,5,6,7,8,9,10 to give the factorization path for the sparse vector B for Fast Forward solution.

A pictorial representation for the sparse vector with 4 nonzeros as discussed is shown in Fig. 2.3. The path for node 1 is formed by tracing a path from this node in ascending node sequence, to the last node (node 10 in this case). The partial paths emanating from nodes 3 and 6 are traced out similarly, until they meet the already formulated path at a junction (node 8).

No additional path is required to be generated for node 7, as it is already contained in the path originating from node 3. The graph of Fig. 2.3 containing the union of

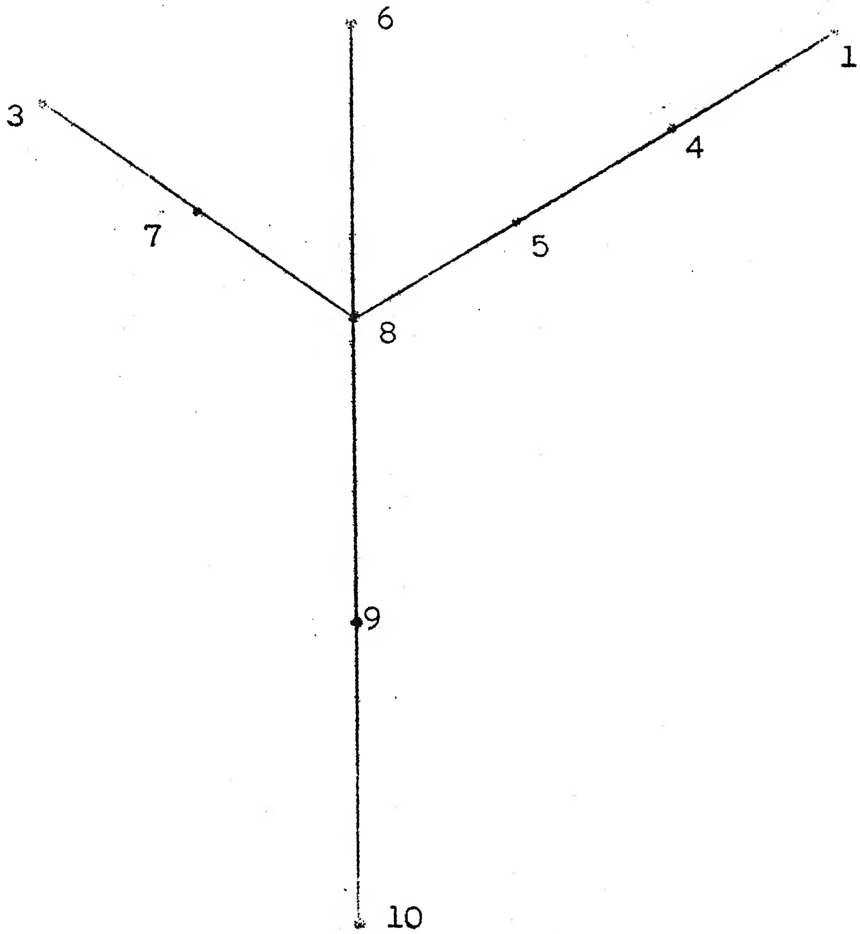

Fig. 2.3: Pictorial Representation or graph for the example 10 node system.

these path segments describes the subset of columns of [L'] for FF solution. An analogous approach would be valid for obtaining the subset of rows of [U] for FB solution.

2.3 TEST EXAMPLES:

To illustrate the sparse vector approach described in the previous section, the solution of a system of linear algebraic equations corresponding to 20 node, 40 node and 80 node synthetic network is obtained employing

a) Fast Forward and Fast Backward solution

b) Full Forward and Full Backward solution

Computer programs have been developed for both the solution methods. In the programs developed, full storage of A matrix is contemplated and as such optimal ordering is not attempted. The effectiveness of the sparse vector solution approach for the various sample systems is investigated through

i) variation in the number and location of nonzero elements in the independent vector B.

ii) Variation in the degree of sparsity of the system matrix A.

With the variation in the number of nonzero elements in the vector B, the computation time requirement for the solution of a system of linear algebraic equations corresponding to a 20 node, 40 node and 80 node network is tabulated in Table 2.1. The computation time reported includes the time for the factorization of matrix A.

It can be seen from Table 2.1 that the computation time for FF/FB solution is largely dependent on the number of nonzeros in the B vector. This fact is particularly obvious in the solution of the 80 node system. Furthermore, for the three cases studied, with large number of nonzero entries in the B vector, the Full Forward/Full Backward solution is more beneficial. This is because of the fact that as the sparsity of vector B decreases i.e. as the number of nonzero elements in vector B increases, the path finding algorithm becomes increasingly inefficient, thereby accounting for the increased computing time of FF/FB solution method.

However, when the B vector is reduced to a singleton, the solution is obtained faster with the FF/FB approach. This observation too is very evident from Table 2.1, in all the three cases. Moreover, for a singleton B, reduction in execution time for FF/FB solution is quite appreciable

Table 2.1: Computational Timings for Solution of 20 Node, 40 Node, and 80 Node System Equations.

	20 NODE System (System Matrix [A] has 20% nonzero elements)			40 NODE System (System Matrix A has 12% nonzero elements)			80 NODE System (System matrix A has 13% nonzero elements)		
Sl. No.	No. of nonzeros in vector B	CPU Time (in seconds)		No. of nonzeros in vector B	C.P.U. time (in seconds)		No. of nonzeros in vector B	C.P.U. time (in seconds)	
		FF/FB solution	Full Forward/Full Backward solution		FF/FB solution	Full Forward/Full Backward solution		FF/FB solution	Full Forward/Full Backward solution
1	20	0.25	0.22	40	2.56	0.84	80	37.13	4.26
2	16	0.25	0.19	32	1.89	0.85	64	25.50	3.88
3	14	0.24	0.20	28	1.62	0.83	56	20.12	3.90
4	8	0.22	0.19	16	1.04	0.82	32	9.56	3.89
5	4	0.24	0.19	8	0.86	0.86	16	5.43	3.92
6	2	0.24	0.23	4	0.84	0.83	8	4.30	3.94
7	1	0.20	0.21	1	0.79	0.82	2	3.85	3.88
8	-	-	-	-	-	-	1	3.80	3.88

as the size of the system is increased. For a smaller system, the Full Forward/Full Backward solution is more relevant. It may be worth mentioning that case studies reported in [3,5], to illustrate the effectiveness of sparse vector method have been carried out on systems having 400 to 2000 nodes.

During the course of the study in Table 2.1 it was also noticed that the variation in the location of the nonzero element, in a singleton B did not have any significant effect on the computation time. A similar observation was also made when the magnitude of nonzero element was changed, keeping its location fixed.

The effect of variation of degree of sparsity of the matrix A, on the execution time for 40 node and 80 node system is shown in Table 2.2. The sparsity of matrix A has been reduced to 5% in both these cases.

A comparison of Table 2.2 with Table 2.1 for the 40 node system reveals that with the reduction in the sparsity of [A], the time required for FF/FB solution reduces drastically, while there is no appreciable change for the time required for the Full Forward/Full Backward solution, even with large number of nonzeros in vector B. The reduction

Table 2.2: Effect of Variation of Degree of Sparsity on 40 Node and 80 Node Systems.

	40 NODE SYSTEM (System Matrix A has 5% nonzero elements)			80 NODE SYSTEM (System Matrix A has 5% nonzero elements)		
		C.P.U. Time (in seconds)			C.P.U. Time (in Seconds)	
Sl. No.	No. of nonzeros in vector B	FF/FB solution	Full Forward/ Full Backward solution	No. of nonzeros in vector B	FF/FB solution	Full Forward/ Full Backward solution
1	40	0.74	0.78	80	4.27	4.26
2	32	0.72	0.78	64	4.30	4.21
3	28	0.72	0.78	56	4.28	4.23
4	16	0.71	0.76	32	4.27	4.24
5	8	0.71	0.76	16	4.25	4.28
6	4	0.70	0.77	8	4.22	4.22
7	1	0.69	0.72	2	4.19	4.22
8	-	-	-	1	4.17	4.30

in the nonzero elements in the A matrix leads to reduction in the path length, consequently resulting in faster computing time. A similar trend is also observed when the 80 node systems of Tables 2.1 and 2.2 are compared. In general, in both cases, FF/FB solution may be preferred when vector B is highly sparse.

Computer programs developed for the solution using FF/FB approach are also capable of solving a system of equations for different independent vectors B. The program avoids the need for refactorization of matrix A, with every change in B vector.

Table 2.3: Comparison of FF/FB Approach and Full Forward/ Full Backward Approach for a 80 Node System. (System Matrix A has 5% nonzero elements).

Sl. No.	No. of nonzeros	Execution Time (in msecs)	
		FF/FB solution	Full Forward/Full Backward solution
1	80	368	141
2	56	241	141
3	2	116	146
4	1	105	169
5	1	98	141

The time requirement for both the solution approaches is recorded in Table 2.3. The execution time shown does not include the time for factorization of A. Once again, it becomes obvious from the results of Table 2.3, that with a small number of nonzero elements in vector B, FF/FB solution method is preferable.

2.4 CONCLUSION:

This chapter reports on the use of sparse vector approach for the solution of a system of linear algebraic equations. The effectiveness of the FF/FB solution method employed in the sparse vector approach is compared to the normal Full Forward/Full Backward solution method, through various case studies. It is observed that the sparse vector methods can be preferred in situations where the system under study is extremely large and the independent vector is highly sparse, if not a singleton.

CHAPTER 3

CONTINGENCY EVALUATION USING SPARSE VECTOR METHOD

3.1 GENERAL:

With the growing complexities of the present day power systems, security analysis has become an important aspect. One of the functions of security analysis which is commonly known as contingency evaluation is to determine whether the normal system is secure or insecure. Based on this information, the preventive action to be taken is determined. In order to ensure satisfactory system operation, contingency evaluation is generally carried out in real time. In view of this, it is important that the techniques for contigency evaluation should be computationally efficient. In this context, this chapter describes the application of sparse vector methods to contingency evaluation.

3.2 CONTINGENCY EVALUATION:

A power system generally undergoes contingencies like outage of transmission line, outage of a transformer or loss of generation etc. Investigation of the effects of these outages is essential both during system planning and operation. Outages may be grouped into two categories.

a) Network outages

b) Power outages

A network outage involves changes in the network admittance parameters and includes the following

i) outage of a transmission line

ii) outage of a transformer

iii) outage of a shunt capacitor or reactor

A power outage brings about changes in bus power injections and includes the following

i) Loss of generating unit

ii) Sudden loss of a load

iii) Sudden change in flow in an inter tie.

It is not practicable to test a power system for every conceivable contingency. Based on historical records and operating experience, a nominal contingency list is prepared. These contingencies are then processed in an order that takes into account their probability and severity. However, it is has been observed that of all the contingencies, the transmission line outages are quite common and thus attention is confined in this chapter to network contingencies.

Network outages result in a change of system configuration. One of the obvious ways for determining the effect

of these contingencies is to carry out fresh load flow for the changed system configuration. But carrying out complete load flow for every contingency is both time consuming and costly, especially if many cases are to be studied. The alternative is to simulate the effect of these outages to obtain corrections, which may be used to update the base case load flow solution, to represent the post contingency operating state of the system.

Several methods for contingency evaluation based on linearized a.c. load flow, distribution factors, d.c. load flow, full a.c. load flow etc. are being used by various power utilities [8-11,13-15]. Fast Decoupled Load Flow [8] provides a straightforward method for contingency evaluation. An iteration cycle of Fast Decoupled load flow consists of a phase angle iteration followed by a voltage magnitude iteration. After obtaining the base case load flow for the normal system, each line outage case in the list of contingencies is then processed by the compensation method using a 'θ' iteration to detect probable overloads. If voltages are a problem, then 'θ' iteration is followed by a 'V' iteration.

Based on Fast Decoupled load flow method, the base case load flow is obtained using the following equations.

$$[B'] [\Delta\theta] = [\frac{\Delta P}{|V|}] \tag{3.1}$$

$$[B''] [\Delta|V|] = [\frac{\Delta Q}{|V|}] \tag{3.2}$$

where,

$[\frac{\Delta P}{|V|}]$ is a vector of real power mismatches

$[\frac{\Delta Q}{|V|}]$ is a vector of imaginary power mismatches

$[\Delta\theta]$ and $[\Delta|V|]$ are solution vectors for phase angle corrections and bus voltages corrections.

Matrices [B'] and [B''] are derived from Y Bus matrix.

Both [B'] and [B''] are real and sparse. Since they contain only network admittances, they are constant and need to be evaluated **once** only at the beginning of the study.

Network outages like line or transformer outages can be simulated using the Inverse Matrix Modification Lemma [5]. Outages are reflected in base case matrices B' and B'' by modifying the affected elements in row k and row m. For each line or transformer outage, only four elements of the matrix B' and B'' undergo change. Without actually modifying B' and B'' and then performing a repeat load flow, a

row vector M which simulates the effect of disconnected line or transformer is introduced. In case, the line 'km' between the kth and mth nodes gets disconnected, the row vector M has the following structure.

$$\begin{array}{cccccccccccccccccc} & & & & \text{kth column} & & & & & & & & \text{mth column} & & & & \\ \underline{M} = [& 0 & 0 & 0 & a & - & - & - & 0 & - & - & - & 0 & 0 & -1 & 0 & 0 & 0 \;] \end{array} \quad (3.3)$$

where,

a = off-nominal turns ratio of the transformer between buses k and m

= 1, for a line

The matrix B_1' corresponding to the post-contingency configuration of the system, consequent to the outage, is obtained as

$$[B_1'] = [B'] + b. \; [M^T][M] \quad (3.4)$$

where,

b = line or transformer series admittance between nodes k and m

Then,

$$[B_1']^{-1} = \{[B'] + b \; [M^T][M]\}^{-1} \quad (3.5)$$

Using the Sherman-Morrison formula [12] and expanding Eqn. (3.5), we have

$$[B_1']^{-1} = [B']^{-1} - b\left\{1+b[M][B']^{-1}[M^T]\right\}^{-1}\left\{[B']^{-1}[M^T][M][B']^{-1}\right\}$$

or,

$$[B_1']^{-1} = [B']^{-1} - b\left\{b\left(\frac{1}{b}+[M][B']^{-1}[M^T]\right)\right\}^{-1}[B']^{-1}[M^T][M][B']^{-1}$$

or,

$$[B_1'] = [B']^{-1} - \left\{\left(\frac{1}{b}+[M][B']^{-1}[M^T]\right)\right\}^{-1}\left\{[B']^{-1}[M^T][M][B']^{-1}\right\}$$

(3.6)

Let $\underline{X} = [B']^{-1}\,\underline{M}^T$ (3.7)

Substituting Eqn. (3.7) in Eqn. (3.6)

$$[B_1']^{-1} = [B']^{-1} - \left\{\frac{1}{b} + \underline{M}\,\underline{X}\right\}^{-1}\underline{X}\,\underline{M}[B']^{-1} \quad (3.8)$$

Replacing $\left\{\frac{1}{b} + \underline{M}\,\underline{X}\right\}^{-1} = C$, Eqn. (3.8) becomes

$$[B_1']^{-1} = [B']^{-1} - C\,\underline{X}\,\underline{M}\,[B']^{-1} \quad (3.9)$$

Phase angle corrections for outages are now derived as under.

Equation (2.1) for the post-contingency state yields

$$[B_1'][\Delta\theta_1] = \left[\frac{\Delta P}{|V|}\right] \quad (3.10)$$

or,

$$[\Delta\theta_1] = [B_1']^{-1}\left[\frac{\Delta P}{|V|}\right] \quad (3.11)$$

Substituting for $[B_1']^{-1}$ from Eqn. (3.9) in Eqn. (3.11)

$$[\Delta\theta_1] = \{[B']^{-1} - C\ \underline{X}\ \underline{M}\ [B']^{-1}\}\ [\frac{\Delta P}{|V|}] \qquad (3.12)$$

or,

$$[\Delta\theta_1] = [B']^{-1}\ [\frac{\Delta P}{|V|}] - C\underline{X}\ \underline{M}\ [B']^{-1}[\frac{\Delta P}{|V|}] \qquad (3.13)$$

But $[B']\ [\frac{\Delta P}{|V|}] = [\Delta\theta]$

Hence $[\Delta\theta_1] = [\Delta\theta] - C\ \underline{X}\ \underline{M}[\Delta\theta]$ (3.14)

Equation (3.14) gives the correction vector as

$$\delta\theta = -C\ \underline{X}\ \underline{M}[\Delta\theta] \qquad (3.15)$$

In exactly similar fashion, bus voltages corrections for the post outage configuration can be computed.

After each solution of Eqn. (3.1), $[\Delta\theta]$ is corrected by an amount

$$\delta\theta = -C\ \underline{X}\ \underline{M}\ [\Delta\theta]$$

Similarly, after each solution of Eqn. (3.2) $[\Delta|V|]$ is corrected by an amount

$$\delta V = -C''\ \underline{X}''\ \underline{M}''\ [\Delta|V|] \qquad (3.16)$$

where superscript '' is used to denote vectors and variables used for obtaining bus voltage corrections.

The technique outlined above, based on Fast Decoupled load flow method can be used for multiple outages as well. For multiple simultaneous outages, the above procedure is applied recursively. The base case solution vector is corrected successively, as the effect of each branch outage is introduced one at a time. For a set of 'n' outages, the recursion is

$$[\Delta\theta_i]=[\Delta\theta_{i-1}]-C_i\underline{X}_i\underline{M}_i[\Delta\theta_{i-1}] \qquad (3.17)$$

$$\text{for } i = 1,2,\ldots n$$

The scalars C_i and the vectors X_i are calculated at the beginning of the case from the recursive algorithm

$$X_i = [B']^{-1} \underline{M}_i^T \qquad (3.18)$$

$$\underline{X}_i = \underline{X}_i - C_j\underline{X}_j\underline{M}_j\underline{X}_i \quad \text{for } j = 1,2,\ldots,(i-1) \qquad (3.19)$$

$$C_i = (1/b_i - \underline{M}_i \underline{X}_i)^{-1} \text{ for all } i=1,2,\ldots n \qquad (3.20)$$

Similar recursive procedure is applied, when bus voltage corrections $[\Delta V]$ are to be computed, for multiple simultaneous outages.

It is of interest to point out that vector M^T in Eqn. (3.7) is highly sparse having only two nonzero elements. The sparsity of this vector can be exploited using sparse vector method in the solution of Eqn. (3.7).

3.3 ALGORITHM FOR CONTINGENCY EVALUATION:

The post contingency state of the system using Fast Decoupled load flow technique is obtained using the algorithm described below. The algorithm proceeds with the assumption that the base case load flow solution is available.

a) B' matrix is obtained from Y Bus matrix in the normal manner.

b) Mismatch vector $\Delta P/|V|$ is computed

c) Row vector M is formed to simulate outages and $\underline{M}^T$ is then obtained. This vector M^T is highly sparse.

d) Vector X is computed using Eqn. (3.7). In obtaining vector X, triangular factors of B' are employed, which are computed once in the beginning.

e) Correction vector $\delta\theta$ is next computed from Eqn. (3.15).

f) Updated $\Delta\underline{\theta}_1$ is then obtained from Eqn. (3.14) and consequently bus angles corresponding to post-contingency state are computed.

g) Analogous steps as in (a) to (f) are also performed for computing updated bus voltages pertaining to the post-contingency state of the system.

Generally one iteration of 'θ' and one iteration of 'V' are performed, as this gives reasonable idea about the system security [13].

3.4 CASE STUDIES:

The application of sparse vector methods in contingency evaluation is illustrated through outage studies on the following systems.

a) IEEE 14 Bus system

b) IEEE 57 Bus system

c) Practical 100 Bus system of a Utility

Computer programs based on Fast Decoupled load flow have been developed for contingency evaluation employing (a) Fast Forward/Fast Backward approach and (b) Full Forward/Full Backward approach for the solution of the system equations.

The computation time for various segments of the algorithm using the two approaches is tabulated in Table 3.1 for one iteration of phase angle correction and one iteration of bus voltage correction.

From Table 3.1, it is evident in all the cases, that while Full Forward/Full Backward approach is faster for computation of uncorrected $\underline{\Delta\theta}$ and uncorrected $\underline{\Delta|V|}$, the FF/FB approach is beneficial for the solution of vectors X and X''. On the basis of this observation, a hybrid approach

Table 3.1: Computational Performance of FF/FB Approach and Full Forward/Full Backward Approach in Contingency Evaluation.

Sl. No.	Program segment	Computation Time (in msecs)					
		IEEE 14 Bus System		IEEE 57 Bus System		100 Bus System	
		Full Forward/ Full Backward	FF/FB	Full Forward/ Full Backward	FF/FB	Full Forward/ Full Backward	FF/FB
1	Uncorrected[$\Delta\theta$]	21	32	851	881	4487	4570
2	Uncorrected[ΔV]	0	9	584	606	2494	2569
3	$\underline{X}$	14	5	819	799	4397	4367
4	$\underline{X}''$	14	10	590	563	2425	2384
5	Complete	298	381	5243	5227	22698	22793

involving Full Forward/Full Backward approach for obtaining uncorrected $\underline{\Delta\theta}$ and uncorrected $\underline{\Delta|V|}$ and FF/FB approach for computing vectors X' and X'' was also examined. The time required for computation in this case is recorded in Table 3.2.

Table 3.2: Analysis of Hybrid Approach in Contingency Evaluation

Sl. No.	Program segment	Computation Time (in msecs)		
		IEEE 14 Bus system	IEEE 57 Bus system	100 Bus system
1	Uncorrected [$\Delta\theta$]	17	854	4480
2	Uncorrected [Δ|V|]	0	586	2499
3	$\underline{X}'$	6	801	4366
4	$\underline{X}''$	9	562	2385
5	Complete	285	5181	22627

Comparison of computation timings in Table 3.1 and Table 3.2 indicates that it would be beneficial to use hybrid approach instead of Full Forward/Full Backward approach or FF/FB approach alone, for contingency evaluation. It is expected that the efficiency of the hybrid approach would be predominantly better than the other approaches, with increase in the system size.

3.5 CONCLUSION:

Contingency evaluation is now one of the very important studies in the planning and operation of a power system. It forms a desirable part of a comprehensive system security monitoring process. As commercial power system utilities these days, evaluate contingency performance of the system very frequently, computationally efficient methods for contingency evolution are greatly sought after. In this chapter, fast decoupled load flow technique exploiting vector sparsity has been examined. It is observed that a hybrid approach involving a judicious mix of FF/FB approach and Full Forward/ Full Backward approach would improve the computational efficiency.

CHAPTER 4

COMPARATIVE STUDY OF COMPENSATION TECHNIQUES FOR CONTINGENCY EVALUATION

4.1 GENERAL:

Compensation methods can be used advantageously for almost any application in which each new case requires a small modification in the network matrix with respect to the base case. Several compensation methods have been reported in the literature [5]. These compensation methods differ from each other in the manner in which compensation is applied. This chapter deals with application of sparse vector approach for contingency evaluation based on different compensation techniques.

4.2 COMPENSATION METHODS:

Compensation methods, which are based on Inverse Matrix Modification Lemma are used to simulate outages and obtain post-contingency state of a system, subjected to outages. A brief outline of the compensation methods is discussed in the following.

In the base case load flow, Fast Decoupled load flow equation for obtaining phase angle correction vector $\Delta\theta$, as already stated in preceding chapter is

$$[B'] \; [\Delta\theta] = [\frac{\Delta P}{|V|}] \tag{4.1}$$

where symbols have their usual meaning.

For changes in the system configuration due to outages, Eqn. (4.1) gets modified to

$$[B_1] \; [\Delta\theta_1] = [\frac{\Delta P}{|V|}] \tag{4.2}$$

The solution of Equation (4.2) gives the post-contingency phase angle as

$$[\Delta\theta_1] = [B_1]^{-1} \; [\frac{\Delta P}{|V|}] \tag{4.3}$$

Applying Inverse Matrix Modification Lemma to Eqn. (4.3) and proceeding in the same manner as for Eqn. (3.5)

$$[\Delta\theta_1] = \left\{[B']^{-1} - C[B']^{-1} \underline{M}^T . \underline{M}[B']^{-1}\right\} [\frac{\Delta P}{|V|}] \tag{4.4}$$

where again symbols used have their usual meaning.
Substituting $X=[B']^{-1} \; M^T$

$$[\Delta\theta_1] = \left\{[B']^{-1} - C \; \underline{X}.\underline{M} \; [B']^{-1}\right\} [\frac{\Delta P}{|V|}] \tag{4.5}$$

Equation (4.4) can be arranged to give different computational arrangements, for determining phase angle correction vector $\Delta\theta_1$. Taking $[B']^{-1}$ over to the right in Eqn. (4.4)

$$[\Delta\theta_1] = \left\{I - C[B']^{-1} \underline{M}^T \; \underline{M}\right\}[B']^{-1} \; [\frac{\Delta P}{|V|}] \tag{4.6a}$$

where I is identity matrix.

Alternatively, moving $[B']^{-1}$ to the left in Eqn. (4.4), vector $\Delta\theta_1$ is obtained as

$$[\Delta\theta_1] = [B']^{-1} \left\{I - C \; \underline{M}^T \; \underline{M}[B']^{-1}\right\} [\frac{\Delta P}{|V|}] \tag{4.6b}$$

Another arrangement is derived when $[B']^{-1}$ is replaced by its factor $[U]^{-1} \; [L]^{-1}$, so as to give

$$[\Delta\theta_1] = [U]^{-1} \left\{ I-[L']^{-1} C\, M^T M [U]^{-1} \right\} [L]^{-1} \left[\frac{\Delta P}{|V|}\right] \tag{4.6c}$$

The different computational arrangements of Eqn. (4.6a), (4.6b) and (4.6c) provide the basis on which different compensation methods have been evolved. Each of these equations can be analysed to have a central calculating step in which network solution is performed by forward and backward substitution, using the sparse [L] and [U] factors of matrix B'. Additionally, there is a compensating step in which compensation for network modification is applied to the system.

Compensation methods are classified depending on the stage at which compensation for network modification is applied to the system. When compensation is provided for, after the network solution is carried out initially, as per Eqn. (4.6a), the method is referred to as post-compensation method. In Pre-compensation, the independent vector $\left[\frac{\Delta P}{|V|}\right]$ is compensated before the network solution as per arrangement of Eqn. (4.6b). Equation (4.6c) gives the equation for Mid-compensation, where the intermediate vector in the middle of the network solution i.e. after the forward solution but before the backward solution, is compensated.

The basic steps involved in the three different compensation methods, for computation of phase angle

corrections are outlined in Table 4.1. Analogous steps would be applicable for determining bus voltage corrections.

It will be seen from Table 4.1 that column vector M^T which is highly sparse is being used for computation of the following vectors.

i) $\underline{X}$, for Post-compensation from equation (4.8)

ii) $\underline{X}_1$, for Pre-compensation from equation (4.13)

iii) $\underline{W}$ and $\underline{W}_1$, for Mid-compensation from equations (4.20) and (4.21) respectively.

This provides the scope for examining the application of sparse vector approach in the compensation methods discussed. In fact, it may be noted that the method used for contingency evaluation in the previous chapter relies on the post-compensation technique. This can be easily inferred by comparing Eqns. (4.5) and (4.6a) with Eqn. (3.12).

4.3 CASE STUDIES:

Case studies reported in the previous chapter dealt with application of sparse vector methods in contingency evaluation based on post-compensation technique. Sparse vector methods may be applied to other compensation methods as well, because of the obvious sparsity structure of $\underline{M}$ and $\underline{M}^T$ vectors. Separate computer programs incorporating

TABLE 4.1

BASIC SOLUTION STEPS FOR POST, PRE AND MID-COMPENSATION

POST-COMPENSATION		
Sl. No.	STEPS	
1	Obtain network solution	
	$\Delta\theta = [B']^{-1} [\frac{\Delta P}{\|V\|}]$	(4.7)
2.	Compute compensation vector $\delta\theta$	
	$\underline{X} = [B']^{-1} \underline{M}^T$	(4.8)
	$Z = \underline{M}\ \underline{X}$	(4.9)
	$C = (1/b + Z)^{-1}$	(4.10)
	$\delta\theta = -C\ \underline{X}\ \underline{M}\ \underline{\Delta\theta}$	(4.11)
3.	Apply compensation	
	$\underline{\Delta\theta}_1 = \underline{\Delta\theta} + \underline{\delta\theta}$	(4.12)

Table 4.1 contd.

PRE-COMPENSATION		
Sl.No.	STEPS	
1	Compute compensating vector (δP)	
	$\underline{X}_1 = [B'^T]^{-1} \; \underline{M}^T$	(4.13)
	$Z_1 = \underline{X}_1^T \; \underline{M}^T$	(4.14)
	$C_1 = (1/b + Z)^{-1}$	(4.15)
	$[\delta P] = -C_1 . \underline{M}^T \; \underline{X}_1^T \; [\frac{\Delta P}{\lvert V \rvert}]$	(4.16)
2.	Apply compensation	
	compensated $[\frac{\Delta P}{\lvert V \rvert}] = [\frac{\Delta P}{\lvert V \rvert}] + [\delta P]$	(4.17)
3	Obtain network solution	
	$\Delta \theta_1 = [B']^{-1} \left\{ \text{compensated } [\frac{\Delta P}{\lvert V \rvert}] \right\}$	(4.18)

Table 4.1 contd.

MID-COMPENSATION		
Sl. No.	STEPS	
1	Start network solution	
	$\underline{F}_1 = [L]^{-1} [\frac{\Delta P}{\lvert V \rvert}]$	(4.19)
2.	Compute compensating vector ΔF	
	$\underline{W} = [L]^{-1} \cdot \underline{M}^T$	(4.20)
	$\underline{W}_1 = (U^T)^{-1} \cdot \underline{M}^T$	(4.21)
	$Z = \underline{W}_1^T \cdot \underline{W}$	(4.22)
	$C = (1/b + Z)^{-1}$	(4.23)
	$\underline{F} = -C.\underline{W}.\underline{W}_1^T\ \underline{F}_1$	(4.24)
3	Apply compensation	
	$\underline{F} = \underline{F}_1 + \quad F$	(4.25)
4	Complete network solution	
	$\underline{\Delta\theta}_1 = [U]^{-1}. \underline{F}$	(4.26)

Fast Forward/Fast Backward (FF/FB) approach and Full Forward/ Full Backward approach were developed to examine the relative efficiency of sparse vector methods in the following compensation techniques.

a) Pre-compensation

b) Mid-compensation

Pre-compensation and mid-compensation have been tested in contingency evaluation, which is based on Fast Decoupled load flow method.

The computation time required for various program segments and the complete program, for Pre-compensation and mid-compensation techniques is shown in Tables 4.2 and 4.3 respectively. In Table 4.2, computation time for Pre-compensation technique has been recorded for the following program segments.

i) Phase angle correction $\underline{\Delta\theta}_1$ from Eqn. (4.18), where the independent vector is non-sparse.

ii) Bus voltage correction $\underline{\Delta V}_1$ using equation analogous to Eqn. (4.18).

iii) Vector $\underline{X}_1$ using Eqn. (4.13), where the independent vector M^T is highly sparse.

iv) Vector $\underline{X}_2$ using equation analogous to Eqn. (4.13).

Table 4.2: Computational Performance of Pre-Compensation Technique in Contingency Evaluation.

Sl. No.	Program segment	Computation Time (in msecs)					
		IEEE 14 Bus System		IEEE 57 Bus System		Practical 100 Bus System	
		Full Forward/ Full Backward	FF/FB	Full Forward/ Full Backward	FF/FB	Full Forward/ Full Backward	FF/FB
1	$\underline{\Delta\theta}_1$ [from Eqn.(4.18)	13	16	832	844	4419	4501
2	$\underline{\Delta V}_1$	4	9	592	599	2434	2563
3	$\underline{X}_1$ [from Eqn.(4.13)	7	6	854	842	4467	4458
4	$\underline{X}_2$	11	9	586	571	2478	2425
5	Complete	369	302	5330	5320	23044	23296

Also, the program segments computed in Table 4.3, for Mid-compensation technique are

i) Vectors $\underline{E}_1$ and $\underline{\Delta\theta}_1$ for phase angle corrections, using equation (4.19) and (4.26) respectively, where the independent vectors are non-sparse.

ii) Vector $\underline{E}_{1V}$ and $\underline{\Delta V}_1$ for bus voltage corrections, using equations analogous to Eqns. (4.19) and (4.26).

iii) Vectors $\underline{W}$ and $\underline{W}_1$ from equations (4.20) and (4.21) respectively, where column vector M^T , is highly sparse.

iv) Vectors $\underline{W}_V$ and $\underline{W}_{1V}$ from equations analogous to Eqns. (4.20) and (4.21).

A study of the computation timings for various program segments in Table 4.2, for Pre-compensation shows that neither of the two approaches i.e. the FF/FB or the Full Forward/Full Backward approach is beneficial by itself. For all the systems studied, FF/FB approach exploiting vector sparsity is advantageous in the solution of equations for highly sparse independent vectors, whereas the Full Forward/Full Backward approach may be preferred, when the independent vector is full or non-sparse. It will be seen that this is in concurrence with the observations made in the preceding chapter, in the context of post-compensation based contingency evaluation.

Table 4.3: Computational Performance of Mid-Compensation Technique in Contingency Evaluation.

Sl. No.	Program segment	(Computation time (in msecs)) IEEE 14 Bus system: Full Forward/ Full Backward	IEEE 14 Bus system: FF/FB	IEEE 57 Bus system: Full Forward/ Full Backward	IEEE 57 Bus system: FF/FB	Practical 100 Bus system: Full Forward/ Full Backward	Practical 100 Bus system: FF/FB
1	$\underline{E}_1$[from Eqn.(4.19)	18	27	821	829	4385	4443
2	$\underline{\Delta\theta}_1$[from eqn.(4.26)	17	25	830	882	4513	4628
3	$\underline{E}_{1V}$	0	0	613	612	2431	2469
4	$\underline{\Delta V}_1$	0	0	584	624	2476	2596
5	$\underline{W}$[from Eqn.(4.20)	18	16	890	850	4519	4404
6	$\underline{W}_1$[fron Eqn.(4.21)	17	13	820	793	4397	4348
7	$\underline{W}_V$	0	0	582	573	2480	2417
8	$\underline{W}_{1V}$	19	15	614	563	2421	2363
9	Complete	392	403	9755	9720	43738	43796

An identical observation can be made from the analysis of computation time requirement recorded in Table 4.3, for contingency evaluation based on Mid-compensation technique. Here again, FF/FB approach is suited for highly sparse independent vectors, while Full Forward/Full Backward approach may be pursued when the independent vector is not highly sparse.

In view of the above observations, again a hybrid approach combining both the approaches i.e. the FF/FB and the Full Forward/Full Backward approach, to exploit their individual advantages, has been explored. The hybrid approach has, been applied for contingency evaluation, based on the three compensation methods under study and the computation timings are reported in Table 4.4.

A comparison of Table 4.4 with Tables 3.1, 4.2 and 4.3, for each of the compensation techniques, brings out that the hybrid approach is more beneficial than either of the other two approaches, when the latter are used in isolation.

It is also possible to make a comparison of the three-compensation techniques as regards the overall computing time from the results of Table 4.4. For the specific case of one outage at a time, the results in Table 4.4 indicate

Table 4.4: Comparative Evaluation of Hybrid Approach in Post,Pre and Mid-Compensation

COMPUTATION TIME (in msecs)							
Sl. No.	Program segment	Post-compensation	Program segment	Pre-compensation	Sl. No.	Program segment	Mid-compensation
IEEE 57 BUS SYSTEM							
1	$\underline{\Delta\theta}$	854	$\underline{\Delta\theta}_1$	831	1	$\underline{E}_1$	819
2	$\underline{\Delta V}$	586	$\underline{\Delta V}_1$	594	2	$\underline{\Delta\theta}_1$	832
					3	$\underline{E}_{1V}$	581
					4	$\underline{\Delta V}_1$	595
3	$\underline{X}$	801	$\underline{X}_1$	841	5	$\underline{W}_1$	849
4	$\underline{X}''$	562	$\underline{X}_2$	565	6	$\underline{W}_1$	797
					7	$\underline{W}_V$	574
					8	$\underline{W}_{1V}$	562
5	Complete	5181	Complete	5311	9	Complete	9590
PRACTICAL 100 BUS SYSTEM							
1	$\underline{\Delta\theta}$	4480	$\underline{\Delta\theta}_1$	4423	1	$\underline{E}_1$	4382
2	$\underline{\Delta V}$	2499	$\underline{\Delta V}_1$	2431	2	$\underline{\Delta\theta}_1$	4517
					3	$\underline{E}_{1V}$	2430
					4	$\underline{\Delta V}_1$	2474
3	$\underline{X}$	4366	$\underline{X}_1$	4453	5	$\underline{W}$	4401
4	$\underline{X}''$	2385	$\underline{X}_2$	2422	6	$\underline{W}_1$	4332
					7	$\underline{W}_V$	2421
					8	$\underline{W}_{1V}$	2354
5	Complete	22627	Complete	23287	9	Complete	43460

that post-compensation seems to be least time-consuming, though Pre-compensation and Post-compensation do not differ significantly in terms of computation time.

However in the cases studied, Mid-compensation takes more time than the other two compensation techniques, though in reference [5], it has been reported that for a general power network problem, Mid-compensation is usually more efficient and faster as compared to the other two compensation approaches.

A direct comparison of the computing time of the compensation methods given in Tables 4.2, 4.3, 4.4 and that reported in [5] is not appropriate. In reference [5] the formulation, has been divided into two phases.

a) The Preparatory Phase - Here the calculation of certain terms of the inverse of matrix B' and B''or their factors, to obtain Z and C is carried out.

b) The Solution Phase - This comprises a standard network solution using the factors of the original unmodified matrices B' and B'', and the compensating steps that introduce the effect of the modification.

In the present case, no distinction has been made in the two phases and the computing effort has been recorded for

the complete formulation. In reference [5], the factors of matrices B' and B'' or their inverses and certain quantities such as C or Z have been assumed to be available in the preparatory phase and then comparison has been made for the computing efficiency of the solution phase for the different compensation methods. Thus, in order to choose the best compensation method for a given application, a clear understanding of compensation fundamentals and efficient implementation is required. Skilful programming of sparse vector and matrix operations is also indispensable.

The results of contingency evaluation for outage of line number 73 for IEEE 57 Bus system for Post, Pre and Mid-compensation based on hybrid approach are tabulated in Table 4.5. These results obtained after one phase angle iteration and one voltage iteration on the base case load flow are quite compatible.

4.4 CONCLUSION:

Compensation methods can be applied advantageously over a wide range of applications, of which contingency evaluation is one such area. In this chapter, application of sparse vector methods has been examined in some of the compensation techniques employed for contingency evaluation.

Table 4.5: Results of Contingency Evaluation for IEEE 57 Bus System

Sl. No.	HYBRID APPROACH					
	Post-Compensation		Pre-Compensation		Mid-Compensation	
	Phase Angle (in degrees) (i)	Bus voltage (in p.u.) (ii)	Phase Angle (in degrees) (iii)	Bus voltage (in p.u.) (iv)	Phase angle (in degrees) (v)	Bus voltage (in p.u.) (vi)
1	0.000000	1.040000	0.000000	1.040000	0.000000	1.040000
2	-1.185225	1.010000	-1.185225	1.010000	-1.185225	1.010000
3	-5.975709	0.985000	-5.975709	0.985000	-5.975709	0.985000
4	-7.322326	0.981916	-7.322326	0.981916	-7.322326	0.981916
5	-8.524521	0.977283	-8.524521	0.977283	-8.524521	0.976850
6	-8.649763	0.998000	-8.649763	0.980000	-8.649763	0.980000
7	-7.585311	0.987353	-7.585311	0.987353	-7.585311	0.985011
8	-4.473096	1.005000	-4.473096	1.005000	-4.473096	1.005000
9	-9.595010	0.980000	-9.595010	0.980000	-9.595010	0.980000
10	-11.472602	0.993533	-11.472602	0.993533	-11.472602	0.989576
11	-10.211752	0.977115	-10.211752	0.977115	-10.211752	0.975804
12	-10.489172	1.015000	-10.489172	1.015000	-10.489172	1.015000
13	-9.823016	0.976852	- 9.823016	0.976852	-9.823016	0.981012
14	-9.379894	0.976866	- 9.379894	0.976866	-9.379894	0.975192
15	-7.199915	0.988482	- 7.199915	0.988439	-7.199915	0.989341
16	-8.871810	1.017442	- 8.871810	1.012580	-8.871810	1.013587
17	-5.402559	1.017442	- 5.402559	1.017442	-5.402559	1.017442
18	-11.512581	1.010950	-11.512581	1.010950	-11.512581	1.010950
19	-13.355300	1.004477	-13.355300	1.004410	-13.355300	1.008345

Table 4.5 Contd..

Sl. No.	(i)	(ii)	(iii)	(iv)	(v)	(vi)
20	-13.804270	1.012948	-13.804270	1.012840	-13.804270	1.020408
21	-12.824954	0.997153	-12.824954	0.996994	-12.824954	1.007935
22	-12.839659	1.002488	-12.839659	1.002318	-12.839659	1.014041
23	-12.901882	1.000393	-12.901882	1.000224	-12.901882	1.012429
24	-13.215916	0.979916	-13.215916	0.979761	-13.215916	0.967064
25	-18.185311	0.957225	-18.185311	0.956971	-18.185311	0.968864
26	-12.922482	0.979642	-12.922482	0.979496	-12.922482	0.982161
27	-11.464569	0.994211	-11.464569	0.994139	-11.464569	0.989936
28	-10.439535	1.006333	-10.439535	1.006289	-10.439535	1.002180
29	- 9.743516	1.018141	- 9.743516	1.018115	- 9.743516	1.018462
30	-18.720272	0.935079	-18.720272	0.934781	-18.720272	0.953649
31	-19.355315	0.902848	-19.355315	0.902444	-19.355315	0.923440
32	-18.398490	0.908646	-18.398490	0.908078	-18.398490	0.935119
33	-18.400300	0.905648	-18.400300	0.905080	-18.400300	0.913445
34	-14.127624	0.958210	-14.127624	0.957488	-14.127624	0.966885
35	-13.879089	0.964204	-13.879089	0.963465	-13.879089	0.973165
36	-13.609742	0.973206	-13.609742	0.972455	-13.609742	0.982292
37	-13.427062	0.981790	-13.427062	0.981380	-13.427062	0.991220
38	-12.724892	1.007916	-12.724892	1.007742	-12.724892	1.017860

Table 4.5: contd.

Sl. No.	(i)	(ii)	(iii)	(iv)	(v)	(vi)
39	-13.471210	0.979861	-13.471210	0.979668	-13.471210	0.988822
40	-13.629890	0.970277	-13.629890	0.969139	-13.629890	0.979197
41	-14.052649	1.006356	-14.052649	1.004988	-14.052649	1.001568
42	-15.529933	0.978217	-15.529933	0.975923	-15.529933	0.973055
43	-11.364676	1.017966	-11.364676	1.017453	-11.364676	1.017643
44	-11.845521	1.012814	-11.845521	1.012673	-11.845521	1.021124
45	- 9.258784	1.033973	- 9.258784	1.033906	- 9.258784	1.037377
46	-11.174955	1.057909	-11.174955	1.057861	-11.174955	1.063733
47	-12.554016	1.030234	-12.554016	1.030136	-12.554016	1.039403
48	-12.632260	1.023757	-12.632260	1.023636	-12.632260	1.029598
49	-12.987981	1.033701	-12.987981	1.033601	-12.987981	1.037687
50	-13.435995	1.021503	-13.435995	1.021431	-13.435995	1.025533
51	-12.545956	1.051520	-12.545956	1.051496	-12.545956	1.051582
52	-11.423458	0.985327	-11.423458	0.985306	-11.423458	0.983537
53	-12.169053	0.974711	-12.169053	0.974693	-12.169053	0.975068
54	-11.662655	0.998206	-11.662655	0.998197	-11.662655	1.000651
55	-10.814055	1.031461	-10.814055	1.031459	-10.814055	1.031469
56	-16.043598	0.980009	-16.043598	0.976774	-16.043598	0.973152
57	-16.559342	0.965700	-16.559342	0.970918	-16.559342	0.965352

Based on the cases studied, it is observed that a hybrid approach having a judicious mix of FF/FB approach for sparse vectors and Full Forward/Full Backward approach for non-sparse vectors, may be more beneficial for network solutions.

CHAPTER 5

CONCLUSION

5.1 GENERAL:

Notwithstanding, the relatively less exploitation of vector sparsity vis-a-vis the matrix sparsity, so far, there exists sufficient basis for gainfully employing sparse vector methods, to further enhance the efficiency of matrix solution methods. Sparse vector approach would be particularly suited in power system applications where the independent vector B is sparse or only a subset of the solution vector X is required to be evaluated. Occurrence of sparse independent vectors is not uncommon in varied power system applications and thus sparse vector methods can lend themselves to utilization in such areas as short-circuit analysis, optimal load flow, economic despatch, contingency analysis, compensation techniques etc.

Normally in the solution of network equations, the Full Forward/Full Backward solution approach is resorted to. The sparse vector method based on the Fast Forward/Fast Backward (FF/FB) approach is examined in Chapter 2, for solution of network equations. In several power system

applications, where the independent vector B is sparse or only a subset of the columns of vector X is to be determined, all nonzero elements are not required in the computation process for obtaining solution of linear algebraic equations. Factorisation paths can be determined using algorithms described in Chapter 2 for identifying the relevant nonzero entries in the sparse A matrix, for the solution of network equations.

A comparison of Full Forward/Full Backward approach with FF/FB approach suggests that Full Forward/Full Backward method may be preferred when the vector B is full or contains large number of nonzero elements. This is because when vector B is predominantly non-sparse, the path finding algorithm becomes time consuming and consequently inefficient. However, the FF/FB approach is faster and hence suitable, when there are very few nonzero entries in vector B or when vector B is a singleton. The computational efficiency of this technique is further enhanced with increasing sparsity of system matrix A.

Having turned out to be effective in the solution of linear algebraic equations, when the independent vector is highly sparse, the viability of sparse vector approach in

power system applications viz. contingency evaluation has been explored. Based on case studies reported in Chapter 3, it may be commented that neither the FF/FB method (i.e. the sparse vector approach) nor the Full Forward/Full Backward approach is beneficial by itself. A hybrid approach integrating FF/FB method with Full Forward/Full Backward method,exploiting their individual merits has proved to be more effective.

Compensation techniques are well suited for contingency evaluation. In Chapter 4,investigation of three compensation techniques namely Post, Pre and Mid-compensation in contingency evaluation, with a view to determining the feasibility of sparse vector approach suggests that the sparse vector approach is capable of improving the computational efficiency of these compensation methods.

Case studies on various sample power systems seem to corroborate the observation that the hybrid approach would be more competitive, than either the FF/FB or the Full Forward/ Full Backward method. Moreover, though the relative effectiveness of these compensation techniques in contingency evaluation has also been analysed, a detailed study of this aspect would be necessary before coming to any definite conclusions, in this regard.

5.2 SCOPE FOR FURTHER WORK:

While the application of sparse vector approach to systems upto 100 buses has been examined, it is important to examine the effectiveness of this approach over larger power system networks.

The effectiveness of the sparse vector approach also needs to be investigated when sparsity oriented storage schemes are employed for sparse matrix A and optimal ordering is resorted to.

It would also be relevant to study the effectiveness of the hybrid approach for contingency evaluation, in greater depth over large systems, considering multiple simultaneous outages.

REFERENCES

1. W.F. Tinney and J.W. Walker, 'Direct Solution of Sparse Network Equations by Optimally Ordered Triangular Factorization', Proc. IEEE, vol. 55, pp. 1801-1809, Nov. 1967.

2. W.F. Tinney, 'Compensation Methods for Network Solutions by Optimally Ordered Triangular Factorization', IEEE Trans. Power App. Syst., vol. PAS-91, pp. 123-127, Jan./Feb. 1972.

3. W.F. Tinney, V. Brandwajn and S.M. Chan, 'Sparse vector Methods', IEEE Trans. on Power Apparatus and Systems, vol. PAS-104, No. 2, Feb. 1985.

4. V. Brandwajn and W.F. Tinney, 'Generalized Method of Fault Analysis', paper submitted to IEEE Trans. on Power Apparatus and Systems, Aug. 1983.

5. O. Alsac, B. Stott and W.F. Tinney, 'Sparsity-oriented compensation methods for modified network solution', IEEE Transaction on Power Apparatus and Systems, vol. PAS-102, pp. 1050-1060, May,1983.

6. D. Sun, B. Ashley, B.A. Hughes and W.F. Tinney, 'Optimal Power Flow by Newton Approach', Paper 84W MO44-4, IEEE Winter Power Meeting, Dallas, Tx, Feb. 1984.

7. K.W. Edwin and G.G. Wagner, 'On-line Security Constrained Economic Dispatch using Sensitivities and Convex Linear Programming', IFAC Symp. on Computer Applications in Large Power Systems, New Delhi, India, Aug. 1979.

8. B. Stott and O. Alsac, 'Fast Decoupled Load Flow', IEEE Trans. PAS, vol. 93, pp. 859-869, May/June 1974.